Non-Leptonic B Decays into K-Resonances

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ABSTRACT

We estimate the non-leptonic B decays $B \to (\psi, \psi', \chi_{1c}) + K^i$, where K^i are various K-meson resonances. We use the model of Isgur, Wise, Scora and Grinstein in the context of heavy quark effective theory, to calculate the hadronic matrix elements. Our estimates show that a substantial fraction of $B \to X_s\psi$ results in higher resonances of K-meson and besides $B \to K(K^*)\psi$, a considerable fraction of $B \to X_s(c\bar{c})$ goes to $B \to (K, K^*) + (\psi', \chi_{1c})$.

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1 Introduction

Recently, there has been some progress in the measurement of some B decays to final states containing strange quarks [1, 2]. The accurate measurement of rare B decays can shed some new light on the Standard Model as well as the physics beyond it. On the other hand, nonleptonic B decays like $B \to K_s J/\psi$ could prove to be important for CP violation [3]. It is the latter processes that are the main focus of this paper.

The CKM favored nonleptonic B decays have been extensively investigated in the literature. In fact, the two body decay $B \to K^{(*)} J/\psi$ is used for the measurement of the B life time [4]. On the other hand, the same process could generate a significant background to the rare decay $B \to K^* \gamma$ through $J/\psi - \gamma$ conversion [5]. Also, a match between theoretical estimate and experimental value of this decay could be used as a check for spectator model.

The main source of uncertainty in the theoretical calculations, is the evaluation of the hadronic matrix element. In a previous paper on this subject, we used the new symmetries in the heavy quark limit to calculate these matrix elements in terms of a single universal Isgur-Wise function (to the leading order in heavy quark expansion) [6]. In that paper, we concluded that indeed this method could be used for an estimate of these processes even though the **s** quark is not particularly heavy.

On the other hand, due to the large mass difference between B meson and $J/\psi, \psi'$andχ_{1c} there is a significant exclusive decay channels to higher K meson resonances. Besides $B \to K^i J/\psi(\psi')$ processes, $B \to K^i \chi_{1c}$ channels must have a sizable fraction indicated by the inclusive measurement $Br(B \to X_s \chi_{1c}) = 0.54\%$, which is even larger than $Br(B \to X_s \psi') = 0.30\%$. The fact that the experimental data on these exclusive decays will be available soon has been the motivation of this work. In this paper, we use the heavy quark symmetries to calculate the hadronic matrix elements relevant to B decays to various K meson resonances. Also, the model of Isgur, Wise, Scora and Grinstein [7] is used for the evaluation of the universal functions.

In section 2, we write down the effective Hamiltonian relevant to CKM favored nonleptonic two body decays of B meson followed by the calculation of the decay rates in terms of the Isgur-Wise functions. Section 3 is devoted to evaluation of these universal functions using IWSG model leading to our estimates for various branching ratios. We end this paper with concluding remarks in section 4.

2 Effective Hamiltonian and Decay Rates

We start with the effective Hamiltonian relevant to $B \to K^i \psi(\psi', \chi_{1c})$ processes which, assuming factorization, can be written as [6, 8]:

$$H_{eff} = C f_{\psi,\psi',\chi_{1c}} \bar{s} \gamma_\mu (1-\gamma_5) b \epsilon^\mu_{\psi,\psi',\chi_{1c}}. \tag{1}$$

where

$$f_{\psi,\psi',\chi_{1c}} \epsilon^\mu_{\psi,\psi',\chi_{1c}} =< 0|\bar{c}\gamma^\mu c|\psi, \psi', \chi_{1c} >,$$

$$C = \frac{G_F}{\sqrt{2}}(c_1 + c_2/3) V^*_{cs} V_{cb}. \tag{2}$$

c_1 and c_2 are QCD improved Wilson coefficients:

$$\begin{aligned} c_1 &= \frac{1}{2}\left(\left[\frac{\alpha_s(\mu)}{\alpha_s(m_W)}\right]^{-6/23} - \left[\frac{\alpha_s(\mu)}{\alpha_s(m_W)}\right]^{12/23}\right), \\ c_2 &= \frac{1}{2}\left(\left[\frac{\alpha_s(\mu)}{\alpha_s(m_W)}\right]^{-6/23} + \left[\frac{\alpha_s(\mu)}{\alpha_s(m_W)}\right]^{12/23}\right). \end{aligned} \tag{3}$$

In the limit where **b** and **s** quarks are considered heavy, matrix elements of (1) are calculated by taking a trace:

$$\begin{aligned} &< K^i(v')|\bar{s}\gamma_\mu(1-\gamma_5)b|B(v) >= \\ &Tr\left[\bar{\Re}^i(v')\gamma_\mu(1-\gamma_5)\Re(v)M(v,v')\right]. \end{aligned} \tag{4}$$

$\Re^i(v')$ and $\Re(v)$ are the matrix representations of K^i and B respectively, v and v' are velocities of initial and final state mesons [9]. M which represents the light degrees of freedom is related to Isgur-Wise functions. In our previous paper, heavy quark symmetries have been applied to the exclusive $B \to (K, K^*) + (\psi, \psi')$ decays, in which the universal function has been determined by the best fit to semileptonic decay data [6]. In order to apply the same method when B decays to higher excited states of K meson, we classify these resonances into spin doublets [5, 12]. Consequently, the decay to each spin doublet, to the leading order in heavy quark expansion, is expressed in terms of a single Isgur-Wise function. The decay rates, obtained in this way, can be written as follows:

$$
\begin{aligned}
\Gamma(B \to K^i \psi, \psi', \chi_{1c}) &= \frac{C^2 f^2_{\psi,\psi',\chi_{1c}}}{16\pi} g(m_B, m_{K^i}, m_{\psi,\psi',\chi_{1c}}) m_{K^i} F_i(x_i) |\xi_I(x_i)|^2, \\
g(m_B, m_{K^i}, m_{\psi,\psi',\chi_{1c}}) &= \left[\left(1 - \frac{m^2_{\psi,\psi',\chi_{1c}}}{m_B^2} - \frac{m^2_{K^i}}{m_B^2} \right)^2 - \frac{4 m^2_{K^i} m^2_{\psi,\psi',\chi_{1c}}}{m_B^4} \right]^{1/2}, \qquad (5) \\
x_i = v.v' &= \frac{m_B^2 + m^2_{K^i} - m^2_{\psi,\psi',\chi_{1c}}}{2 m_B m_{Ki}}.
\end{aligned}
$$

The Isgur-Wise functions for each spin doublet $\xi_I(x)$, $I = C, E, F, G$ are labeled following reference [12]. In table 1, the functions $F_i(x)$ for various excited states of K meson are tabulated.

On the other hand, using (1) we can obtain the inclusive decay rate $\Gamma(B \to X_s \psi, \psi', \chi_{1c})$ which is taken to be equal to $\Gamma(b \to s\psi, \psi', \chi_{1c})$:

$$
\begin{aligned}
\Gamma(b \to s\psi, \psi', \chi_{1c}) &= \frac{C^2 f^2_{\psi,\psi',\chi_{1c}}}{8\pi m_b m^2_{\psi,\psi',\chi_{1c}}} g(m_b, m_s, m_{\psi,\psi',\chi_{1c}}) \times \\
&[m_b^2(m_b^2 + m^2_{\psi,\psi',\chi_{1c}}) - m_s^2(2m_b^2 - m^2_{\psi,\psi',\chi_{1c}}) + m_s^4 - 2m^4_{\psi,\psi',\chi_{1c}}], \qquad (6)
\end{aligned}
$$

where ψ, ψ'andχ_{1c} in these decays are directly produced. Consequently, the experimental knowledge of these inclusive decays yields the branching ratio for various exclusive channels if the Isgur-Wise functions are known.

3 IWSG Model for Calculating Isgur-Wise Functions

To obtain a numerical estimate of the branching ratios, we have to insert in (5) the Isgur-Wise functions evaluated at x_i. These functions which represent the nonperturbative QCD effects are a measure of light cloud (spectator quark) rearrangement around the heavy quark during the weak transition. There are various models for determining these universal functions from which we use the wavefunction model of Isgur, Wise, Scora and Grinstein [7]. In this model, the functions ξ_I can be obtained from the overlap integrals

$$
\xi(v.v') = \sqrt{2L+1}\, i^L \int r^2 dr \Phi_F^*(r) \Phi_I(r) j_L \left[\Lambda r \sqrt{(v.v')^2 - 1} \right], \qquad (7)
$$

where I and F refer to the radial wavefunction of the initial and final state mesons respectively, L is the orbital angular momentum of the final state meson and j_L is the spherical Bessel function of order L. The inertia parameter is taken to be [10]

$$
\Lambda = \frac{m_{K^i} m_q}{m_s + m_q},
$$

where m_q is the light quark mass.

To calculate (7), we follow IWSG model in using harmonic oscillator wavefunctions with oscillator strength β for the radial wavefunctions Φ_I and Φ_F. Note that we assume β to be the same for initial and final state mesons which is necessary if the normalization condition $\xi_C(1) = 1$ and $\xi_{E,F,G}(1) = 0$ are to be satisfied. For example, using the ground state radial wavefunction

$$\Phi(r) = \frac{\beta^{3/2}}{\pi^{3/4}} exp\left[-1/2\beta^2 r^2\right],$$

one obtains

$$\xi_C(v.v') = exp\left[-\frac{9}{256\beta^2} m_{K^i}^2 (1 - (v.v')^2)\right]. \tag{8}$$

We fix a value for β by the best fit of (8) to experimentally measured $B \to (K, K^*) + (\psi, \psi')$ decays [2]. This leads to $\beta = 0.295 GeV$ which is smaller than those values fitted to the semileptonic B and D decays i.e., $\beta_K = 0.34$ and $\beta_B = 0.41 GeV$ [7]. In our previous paper [6], we used the same best fit of the Isgur-Wise fuction to estimate the exclusive decay rate for $B \to K^* + \gamma$. The result is in reasonable agreement with the recent measurment reported by CLEO II [1]. Inserting the resulting values for the Isgur-Wise functions in (5) and using the following experimental input [2]

$$BR(B \to X_s \psi) = (1.09 \pm 0.04 \pm 0.01)\%,$$

$$BR(B \to X_s \psi') = (0.30 \pm 0.05 \pm 0.03)\%,$$

and [13]

$$BR(B \to X_s \chi_{1c}) = (0.54 \pm 0.21)\%,$$

lead to our estimate of the branching ratios reflected in tables 2, 3 and 4. The mass parameters that appear in our calculations are taken as (in GeV)

$$\begin{array}{ll} m_q = 0.33 & m_s = 0.55 \\ m_b = 4.9 & m_B = 5.28 \\ m_\psi = 3.10 & m_{\psi'} = 3.69 \\ m_{\chi_{1c}} = 3.51 & \end{array}$$

4 Conclusion

We observe from Table 2 that a substantial fraction of $B \to X_s \psi$ decays result in higher K resonances other than K and K^*. For example, B decay to $K_2^*(1430)$ is comparable to its decay to $K^*(892)$. This is not the case, more for $B \to X_s \psi'$ and to a lesser extent for $B \to X_s \chi_{1c}$, mainly due to phase space suppression. A large branching ratio for

the $K_2^*(1430)$ channel in the radiative rare B-decays is also pointed out by authors of refs. [10, 11, 12]. For $B \to K(K^*) + (c\bar{c})$ prosesses, the χ_{1c}-channel gives a considerable contribution. For example, $Br(B \to K^* + \chi_{1c})$ is even larger than $Br(B \to K^* + \psi')$. Thus, for the sake of increasing statistics, $B \to K^i + \chi_{1c}$ channels are also favored. The test of these theoretical predictions should be within experimental reach very soon. Finally, we would like to emphasize that even though **s** quark is not particularly heavy, our results can be taken as an order of magnitude estimate.

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K^i Name	J^P	$F_i(x)$
$K(498)$	0^-	$(x+1)[(x+1)(m_B - m_K)^2/m_\psi^2 - 2]$
$K^*(892)$	1^-	$(x+1)[(x-1)(m_B + m_{K^*})^2/m_\psi^2 + 4x + 2]$
$K^*(1430)$	0^+	$(x-1)[(x-1)(m_B + m_{K^*})^2/m_\psi^2 + 2]$
$K_1(1270)$	1^+	$(x-1)[(x+1)(m_B - m_{K_1})^2/m_\psi^2 + 4x - 2]$
$K_1(1400)$	1^+	$2/3(x-1)(x+1)^2[(x+1)(m_B - m_{K_1})^2/m_\psi^2 + x - 2]$
$K_2^*(1430)$	2^+	$2/3(x-1)(x+1)^2[(x-1)(m_B + m_{K_2^*})^2/m_\psi^2 + 3x + 2]$
$K^*(1680)$	1^-	$2/3(x+1)(x-1)^2[(x-1)^2(m_B + m_{K^*})^2/m_\psi^2 + x + 2]$
$K_2(1580)$	2^-	$2/3(x+1)(x-1)^2[(x+1)(m_B - m_{K_2})^2/m_\psi^2 + 3x - 2]$
$K(1460)$	0^-	$(x+1)[(x+1)(m_B - m_K)^2/m_\psi^2 - 2]$
$K^*(1410)$	1^-	$(x+1)[(x-1)(m_B + m_{K^*})^2/m_\psi^2 + 4x + 2]$

Table 1. $F_i(x)$ for various K-meson excited states.

K^i Name	J^P	Mass (MeV)	$x_\circ$	$\xi_I(x_\circ)$	$BR(B \to K^i\psi)$
K	0^-	497.67 ± 0.03	3.52	0.319	0.094
$K^*(892)$	1^-	896.1 ± 0.3	2.02	0.368	0.227
$K^*(1430)$	0^+	1429 ± 7	1.35	0.631	0.026
$K_1(1270)$	1^+	1270 ± 10	1.48	0.626	0.075
$K_1(1400)$	1^+	1402 ± 7	1.37	0.630	0.087
$K_2^*(1430)$	2^+	1425.4 ± 1.3	1.35	0.631	0.202
$K^*(1680)$	1^-	1714 ± 20	1.17	0.326	0.001
$K_2(1580)$	2^-	≈ 1580	1.24	0.364	0.003
$K(1460)$	0^-	≈ 1460	1.32	-0.275	0.014
$K^*(1410)$	1^-	1412 ± 12	1.36	-0.282	0.088
					TOTAL: 0.817

Table 2: The branching ratios for B decay to ψ and various K mesons.

K^i Name	J^P	Mass (MeV)	$x_\circ$	$\xi_I(x_\circ)$	$BR(B \to K^i \psi')$
K	0^-	497.67 ± 0.03	2.76	0.515	0.052
$K^*(892)$	1^-	896.1 ± 0.3	1.59	0.609	0.180
$K^*(1430)$	0^+	1429 ± 7	1.08	0.433	0.001
$K_1(1270)$	1^+	1270 ± 10	1.18	0.538	0.005
$K_1(1400)$	1^+	1402 ± 7	1.10	0.466	0.002
$K_2^*(1430)$	2^+	1425.4 ± 1.3	1.08	0.432	0.004
$K^*(1680)$	1^-	1714 ± 20	0.950	-	-
$K_2(1580)$	2^-	≈ 1580	1.00	-	-
$K(1460)$	0^-	≈ 1460	1.06	-0.078	< 0.001
$K^*(1410)$	1^-	1412 ± 12	1.09	-0.106	0.003
					TOTAL: 0.247

Table 3: The branching ratios for B decay to ψ' and various K mesons.

K^i Name	J^P	Mass (MeV)	$x_\circ$	$\xi_I(x_\circ)$	$BR(B \to K^i \chi_{1c})$
K	0^-	497.67 ± 0.03	3.01	0.446	0.076
$K^*(892)$	1^-	896.1 ± 0.3	1.73	0.524	0.235
$K^*(1430)$	0^+	1429 ± 7	1.17	0.565	0.004
$K_1(1270)$	1^+	1270 ± 10	1.28	0.605	0.020
$K_1(1400)$	1^+	1402 ± 7	1.18	0.568	0.010
$K_2^*(1430)$	2^+	1425.4 ± 1.3	1.17	0.564	0.032
$K^*(1680)$	1^-	1714 ± 20	1.02	0.053	< 0.001
$K_2(1580)$	2^-	≈ 1580	1.08	0.164	< 0.001
$K(1460)$	0^-	≈ 1460	1.15	-0.172	0.001
$K^*(1410)$	1^-	1412 ± 12	1.18	-0.188	0.017
					TOTAL: 0.395

Table 4: The branching ratios for B decay to χ_{1c} and various K mesons.